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Solitons, branes and black holes

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Abstract : A review on some supersymmetric solitons in string theory and their application to black hole physics is presented.

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In the past two years there has been remarkable progress in understanding the strong coupling behaviour of string theory. These developments have illuminated many long-standing puzzles in string theory and in the physics of black holes. In this talk, I shall review some of these ideas with emphasis on the spectrum of supersymmetric solitons in the theory, which has played an important role in the evolution and application of these ideas. This talk is aimed primarily at an audience of particle physicists, phenomenologists and experimentalists not actively working in string theory.

Much of this progress centers around the notion of 'duality'. Duality is a gauge symmetry that relates a given theory A at strong coupling to another theory B at weak coupling. Complicated strong coupling phenomena in A are related to much simpler weak coupling phenomena in B which can be analyzed using ordinary semiclassical and perturbative techniques. Thus, once we are able to find the dual description of a theory A, many difficult nonperturbative questions that were inaccessible before, become tractable. Duality is a symmetry that requires the knowledge of the full quantum theory and is not apparent in perturbation theory. In this respect it is very different from the usual symmetries in quantum field theory which are true order by order in perturbation theory. Therefore, in order to establish duality, we need to know something about the behaviour of the theory at strong coupling. Duality would not be such a useful symmetry if we needed to know

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everything about the strong coupling behaviour of a theory to find out what it is dual to. One of the main ingredients that makes it possible to assert anything at all about duality without actually solving the strong coupling theory is supersymmetry. Supersymmetry places powerful constraints on the structure of the quantum theory. With enough supersymmetry, many semiclassical quantities receive no quantum corrections-either perturbative or nonperturbative. One can thus learn much about the strong coupling behaviour of a theory by analyzing such quantities in the semiclassical, weak coupling regime and then analytically continuing in the strong coupling regime. In particular, if two theories are dual, then various nonrenormalized quantities should match in the two theories. For example, in theories with sixteen or more supercharges, the spectrum of massless states and their effective action is not renormalized and is completely determined by the gauge symmetry, supersymmetry and low energy consistency conditions such as anomaly cancellation. Therefore two theories cannot be dual unless their effective actions match.

An important observation of Witten and Olive is that this nonrenormalization of the spectrum is valid not only for the massless states but for all so called 'supersymmetric' states. A supersymmetric state, or 'super state' for short, preserves some of the supersymmetries and it follows from the supersymmetry algebra that it saturates a Bogomolnyi bound *i.e.*, its mass M equals the absolute value of some charge Z. The renormalization of its mass is therefore related to the renormalization of the charge. With enough supersymmetries, the charge is sometimes not renormalized, which then implies that the mass is also not renormalized. Another way to characterize super states is that they belong to small representations of the supersymmetry algebra. Recall that the representations of supersymmetry algebra are analogous to the representations of the Poincare algebra. The Poincare algebra can have either massless or massive representations. For higher spin, the dimension of the massless representation is smaller than the dimension of the massive representation. For example, in 3 + 1 spacetime dimensions, the massive vector representation is three dimensional whereas the massless vector representation is two dimensional. This reflects the fact that a massive vector has longitudinal polarization in addition to the two transverse polarizations of the massless vector. A small representation of the supersymmetry algebra which represents a superstate is exactly analogous to the small representation of the Poincare algebra which represents a massless state. The Bogomolnyi inequality $M \geq |Z|$ is the analog of $M \geq 0$. The states which saturate the bound are in a small representation whereas the states which do not are in a large representation. This characterization of superstates explains why one might expect their semiclassical spectrum to be exact. The Bogomolnyi bound $M = |Z|$ relates renormalization of mass to renormalization of charge. Now, even though the mass can vary continuously, the dimension of the representation cannot vary continuously. For the mass to become even slightly larger than $|Z|$, the dimension of the representation would have to jump discontinuously. Unless there are additional nearby states that can fill out a large representation, one would therefore expect the spectrum of superstates to be stable. Moreover, if the charge is not renormalized, the mass also cannot get renormalized. Therefore, the states that are tagged by a particular mass and charges at weak coupling continue to have the same mass and charges even at strong coupling.

One of the simplest and the first example of a duality in four dimensions is in the context of super Yang-Mills theory with gauge group $SU(2)$ with $N = 4$ supersymmetry. In addition to the $SU(2)$ gauge fields, the bosonic fields of the theory include six real scalars transforming in the adjoint of $SU(2)$. The scalar potential is zero because of supersymmetry. The scalars can acquire arbitrary nonzero expectation value so there is a noduli space of the theory parametrized by the scalar expectation value. At the origin of the moduli space, where all scalar expectation values are zero, the theory has the full $SU(2)$ symmetry. Away from the origin the $SU(2)$ is spontaneously broken to $U(1)$. The $N = 4$ theory has four Majorana supercharges or sixteen real supersymmetries. In this case, the small representation for a superstate with spin one is 16-dimensional. The spectrum of superstates of the theory consists of monopoles, dyons and massless and massive vector bosons which all belong to 16-dimensional representations. The charge $|Z|$ in the Bogomolnyi bound is given by $a\sqrt{n^2e^2+m^2e^{-2}}$ where a is the expectation value of the Higgs scalar, e is the coupling constant, n is the electric charge and m is the magnetic charge. This formula can be easily derived in the semiclassical regime and then is exact by earlier arguments. One of the striking features of the mass formula is that it is invariant under $e \leftrightarrow 1/e$ and $m \leftrightarrow m$. This observation led Montonen and Olive to a remarkable conjecture that this theory may be dual to itself. Note that massive vector bosons have $m = 1$ and $n = 0$. Under the duality transformation these would get interchanged with the magnetic monopoles, which have $m = 0$ and $n = 1$. Thus, at weak coupling, when e is small, the fundamental particles of the theory are the vector bosons. At strong coupling, when e is large, the magnetic monopoles are lighter than the vector bosons, and in fact, the theory has dual description in terms of an $N = 4$ super Yang-Mills theory with coupling constant $1/e$ which is weakly coupled; the magnetic monopoles of the original theory are the fundamental particles of the dual theory. One of the main pieces of evidence that has led to the acceptance of this conjecture was provided by the spectrum supersymmetric states. If duality is indeed a symmetry, then it makes precise prediction about the degeneracy of various states. For instance, nonperturbative dyonic states that are related by duality to some perturbative states must have the same degeneracy as the known degeneracy of the dual perturbative states. The degeneracy of dyonic states can be computed using semiclassical techniques. Predictions of duality provide many nontrivial mathematical consistency checks which were verified explicitly for the lowest lying dyons first by Sen. In fact, the entire spectrum of dyonic states in this theory appears to be invariant under duality.

If fundamental particles can arise as solitons in some dual theory, it is natural to wonder if fundamental strings also might arise as solitons in some dual theory, and if string theory may have a dual description in terms of another string theory or some other theory. By analogy with point particle field theory one might expect that the spectrum of superstates in string theory can yield important nonperturbative information. Indeed, during past few years analysis of supersymmetric states in string theory has proved to be extremely fruitful and has provided some of the key insights into the structure of the theory. The expectation that fundamental strings themselves might arise as solitons in some dual theory has now been borne out in several concrete examples. This and several other seemingly unrelated developments have culminated into the concept of duality as a general principle that governs the structure of string theories.

To discuss these ideas concretely, let us consider, for definiteness, heterotic string theory compactified on a six torus. At low energies this theory contains $N = 4$ super Yang-Mills theory with adjoint matter. Therefore the superstates in the full string theory definitely contain the dyonic states of this field theory described earlier. However, the spectrum of superstates is much richer in string theory than in field theory. Even the perturbative superstates are far numerous in string theory. For example, for every Yang-Mills vector boson in the low energy field theory there corresponds an infinite tower of superstates in string theory.

To describe these states, let us assume for simplicity that the internal six-torus is product of six circles, all of radius R. Now, a closed string that wraps around one of this circle is stable because the winding number is topologically conserved. In addition to the winding the closed string can have arbitrary oscillations that travel to the left or to the right along the string. The oscillations of the string can be decomposed in terms of the normal modes. Each normal mode with n nodes is a harmonic oscillator with frequency n, hence a free quantum string is simply a collection of infinite harmonic oscillators with some total left-moving oscillations number N_L and right-moving oscillation number N_R. A state with nonzero oscillations carries nonzero quantized momentum M/R flowing along the circle. The states are thus specified by the winding number L, momentum P, oscillations numbers N_L and N_R and four dimensional mass m. Physical states have to further satisfy the mass-shell conditions as well as gauge constraints :

$$m^2 = (LR + M/R)^2 + (N_L - 1) = (LR - M/R)^2 + N_R.$$

Which of these states are supersymmetric ? In the Green-Schwarz formalism in the light-cone gauge, the heterotic string has eight spacetime supersymmetries, all of which are carried by right-moving currents on the string. The right-moving oscillator ground state therefore preserves all supersymmetries and furnishes a 16-dimensional, spin one, small representation of the supersymmetry algebra. Therefore, if a state is in the right-moving ground state, then it is expected to be supersymmetric. Such a state has no right-moving oscillations ($N_R = 0$), but can have arbitrary left-moving oscillations (N_L arbitrary) subject to the constraint $N_L = 1 - ML$. It saturates a Bogomolnyi bound $M = |LR - N/R|$. Now, as it happens in Kaluza-Klein theories momentum and winding along the internal circle manifest themselves as gauge charges in four dimensions. The state has two charges $q_R = LR - M/R$, $q_L = LR + M/R$, that couple to two abelian gauge bosons with gauge groups $U(1)_L \times U(2)_R$. The states with $N_L = 0$ have no left-moving oscillations and are given by $L = 1$, $M = 1$ and $L = -1$, $M = -1$. These correspond to the massive vector bosons. In fact, when $R = 1$, the mass of these states goes to zero and the gauge symmetry is enhanced to $U(2)$. Changing the radius away from $R = 1$ corresponds to Higgsing the symmetry to break it spontaneously to $U(1)_L \times U(1)_R$.

The number of such supersymmetric string states with a given mass m and given charges q_L and q_R is very large. If we choose the charges such that $N_L = 1 - ML \equiv 1 + \frac{1}{2}(q_R^2 - q_L^2)$ is large, then the number states grows exponentially as $\exp 4\pi\sqrt{N_L}$. This very rapid growth is characteristic of string theory. A given total oscillator number N_L can be obtained many different ways by exciting many different normal modes, say, $n_1, n_2, \cdots, n_k$. The oscillator with frequency n_1 could be in at a level m_1 and so on. The total oscillator number is then $N_L = \sum_{k=1}^{\infty} n_k m_k$, thus the number of states with a given value N_L is simply the number of ways the integer N_L can be written as a sum of smaller integers. For large N_L, it is easy to see that this number goes as $\exp 4\pi\sqrt{N_L}$.

So far I have described these superstates as perturbative string states. There is another way to think about these states which is useful. When the radius R of the circle around which the closed string wraps is large, the string looks like a large macroscopic cosmic string in five dimensions. Now, string theory contains supergravity and super Yang-Mills theory as a low energy limit. Therefore, these macroscopic string states must exist as solutions of the low energy equations of motion. Such a Macroscopic Superstring solution was found by Dabholkar, Harvey, Gibbons and Ruiz. It reveals a rich structure with many parallels in the theory of supersymmetric solitons in field theory. The energy per unit length of this solution saturates a Bogomol'nyi bound and is proportional to some charge per unit length. Consequently some of the supersymmetries remain unbroken in the background of a macroscopic superstring. The remaining broken supersymmetries give rise to chiral fermion zero modes on the string that carry the spacetime supersymmetry charges along the superstring exactly as one would expect for a fundamental chiral superstring. One can also write down the solutions of the low energy action that describe the entire tower of the fundamental string-states described above that carry arbitrary left-moving oscillations and are still supersymmetric.

The macroscopic string solution describes a one-dimensional soliton or a 1-brane. The spectrum of supersymmetric solitons in string theory contains other extended, higher dimensional objects called p-branes. Solutions of the low energy effective action corresponding to various configurations of such branes have now been found. A great deal has since been learnt about various brane-solitons in string theory which together provide overwhelming evidence for the existence of duality as an exact quantum symmetry.

Let me now briefly describe an application of these ideas to the physics of black holes. Let me begin by reviewing the relevant aspects of black holes. Black holes occur as solutions of Einstein equations in vacuum and are widely believed to be the end point of gravitational collapse of a very massive star. Once a black hole is formed, the spacetime develops an event horizon from behind which nothing can escape to observers at asymptotic infinity sitting far away from the black hole. In classical general relativity, a black hole is completely characterized by its mass, charges and angular momentum. Thus, there is no trace of what matter collapsed to form the black hole once it falls behind the event horizon. A classical black hole poses a puzzle in thermodynamics. If we throw into the black hole some matter carrying a lot of entropy like a bucket of hot water, the total entropy of the outside world can be reduced apparently violating the second law of thermodynamics. Quantum mechanically, the black hole radiates by Hawking radiation that is approximately thermal with characteristic temperature called the Hawking temperature that depends only on the mass, charges and angular momentum. Thus a black hole with a given mass will be in thermal equilibrium with thermal radiation bath in a cavity (assumed to be sufficiently small to ensure thermodynamic stability). By the first law of thermodynamics, the black hole must carry entropy. One of the beautiful results in black hole physics is that a black hole carries a huge entropy called the Bekenstein-Hawking entropy which is given exactly by $A_H/4$ where A_H is the area of event horizon in Planck units. With this definition of the black hole entropy, one can formulate a generalized second law of thermodynamics such that the total entropy of a black hole and the external world always increases in any process. This law is found to be true in a number of gedanken experiments. Thus the Bekenstein-Hawking entropy behaves in very respect like ordinary entropy. In statistical mechanics, entropy equals the logarithm of the number of available microstates of a given system with fixed mass *etc.* It is natural to ask what the microstates of the black hole are which can account for its enormous entropy. This has been an outstanding puzzle in black hole physics for almost two decades.

String theory provides for the first time a possible explanation of black hole entropy in terms of counting of microstates for some special kind of black holes. We have seen that the theory contains some supersymmetric states with a very large degeneracy. As I described, there are two ways of thinking about the superstate which are complementary to each other and which describe the same state in different regimes. For a given mass m and charges q_L and q_R which can all be very large, we can choose the string coupling constant λ to be sufficiently small so that all interactions are very weak. One can the ignore the gravitational and electromagnetic field around the state and describe it a perturbative string state in a flat spacetime. If we express N_L in terms of mass and charges and reinstate the string coupling constant, then we find that the number of states with a given mass m and charges q_L and q_R, goes as

$$\exp\left[4\pi\sqrt{N_L}\right] \equiv \exp\left[\frac{8\pi}{\lambda}\sqrt{m^2 - q_L^2}\right].$$

As we increase the coupling constant, keeping the mass and the charges fixed, the effects of gravity and other interactions become large. In particular, the condensate of massless fields around the state can no longer be treated as small. In this regime, it is more appropriate to describe these states as solutions of the low energy supergravity, super Yang-Mills equations of motion. The resulting spacetime is no longer flat and in fact describes the condensate of various massless fields such as gravitons, photons, dilaton *etc* that is formed around the state. We discussed the macroscopic string solution earlier which describes the superstate when the radius of the wrapping circle is large. We can take the radius to be large compared to the string scale but still macroscopically small. From afar, a string that is wrapped around an internal circle looks like a point particle state in four dimensions. In fact, the four dimensional solution corresponds to nothing but a supersymmetric black hole specified by the mass m and charges q_R and q_L. One would therefore expect that the entropy of this black hole should equal the logarithm of the number of perturbative string states with the same quantum numbers. One difficulty in testing this idea is that for supersymmetric black holes which saturate the Bogomolnyi bound, $m = |Z|$, the horizon is singular, *i.e.*, curvature of spacetime near the horizon is large. As a result, defining the area of the horizon is problematic and naively it is zero giving vanishing entropy. Sen argued that massive modes of the string would correct the black hole solution of the low energy effective action black holes. He argued further that the area of surface that is a string-scale away from the horizon, the so called stretched horizon, would give the subleading correction to the entropy. This entropy is indeed found to be in striking agreement with the logarithm of the number of states. It equals $C\sqrt{N_L}$ where C is numerical constant that remains undetermined because we do not know how to incorporate the stringy corrections to the solution. However, it correctly reproduces the nontrivial dependence on three independent parameters λ, m and q_L.

Subsequently, Strominger and Vafa considered, instead of an oscillating fundamental string wrapped around an internal circle, a configuration of 5-branes and a 1-branes in ten spacetime dimensions wrapped on an internal 5-torus which gives a point-like supersymmetric state in the remaining noncompact five spacetime dimensions. The state carries some momentum along the 1-brane and is thus specified by three quantum numbers : Q_1, Q_5 and N, which are the number of 1-branes, the number of 5-branes and the quantized right-moving momentum along the brane, respectively. The advantage of considering this configuration is that when the coupling is large, the resulting black hole has a horizon which is nonsingular and has a nonzero area. Thus, there is no need to calculate the subleading correction to entropy as in the case of black holes considered by Sen. The Bekenstein-Hawking entropy, $A_H/4$, is then found to be $2\pi\sqrt{Q_1 Q_5 N}$. Following earlier logic, the number of superstates does not change as we vary the coupling, so can be computed when the coupling is small. Remarkably, the logarithm of the number of states computed at weak coupling agrees exactly, now even including the numerical constant, with the Bekenstein-Hawking entropy of the corresponding black hole at strong coupling. String theory has thus provided with an example where the properties of the black holes can be understood (at weak coupling) in terms of a collections of various brane solitons. Many other properties of these black holes have been checked since, and the two pictures are found to be in perfect and often surprising agreement.

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